Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Monday 18 October 2021 – Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced
PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 6 8 8 2 4 A 0 1 2 0

Pearson

1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})$ m.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

(a) Method 1 Suvat

~~s~~

$u = 4i\ ms^{-1}$

$v = v$

$a = (2i - 3j)ms^{-2}$

$t = 2$

Use formula $v = u + at$ M1

$v = 4i + 2(2i - 3j)$

$v = 8i - 6j$ A1

Method 2 Integration

To get velocity from acceleration, integrate

$a = (2i - 3j)ms^{-2}$

$v = \int a\,dt = \int 2i - 3j\ dt$

$= 2ti - 3tj + c$ M1

★ Simple Integration: $\int x^n dx = \frac{1}{n}x^{n+1} + c$

Substitute $t=0$, $v = 4i$ to get c:

$4i = 2(0)i - 3(0)j + c$

$4i = c$

$\therefore\ v = (2t + 4)i - 3tj$

Substitute $t = 2$:

$v = (2(2) + 4)i - 3(2)j$

$v = 8i - 6j$ A1

(b) Method 1 Suvat

$s = s$

$u = 4i\ ms^{-1}$

~~v~~

$a = (2i - 3j)ms^{-2}$

$t = 3$

Use formula $s = s_0 + ut + \frac{1}{2}at^2$ M1

$s = (i + j) + 4i(3) + \frac{1}{2}(2i - 3j)(3)^2$

$s = (i + j) + 12i + 9i - \frac{27}{2}j$

$s = 22i + 12.5j$ A1

P 6 8 8 2 4 A 0 2 2 0

Question 1 continued

Method 2 Integration

Integrate velocity to get the displacement function

$$v = (2t+4)i - 3tj$$

$$d = \int v\,dt = \int (2t+4)i - 3tj\,dt$$

$$= (t^2+4t)i - \frac{3}{2}t^2 j + c$$ M1

★ Simple Integration: $\int x^n\,dx = \frac{1}{n}x^{n+1} + c$

Substitute $t=0, d=(i+j)$ to get c:

$$(i+j) = (0^2+4(0))i - \frac{3}{2}(0)^2 j + c$$

$$c = i + j$$

$$\therefore\ d = (t^2+4t+1)i + \left(-\frac{3}{2}t^2+1\right)j$$

Substitute $t=3$:

$$d = (3^2+4(3)+1)i + \left(-\frac{3}{2}(3)^2+1\right)j$$

$$d = 22i - \frac{29}{2}j$$ A1

(Total for Question 1 is 4 marks)

2.

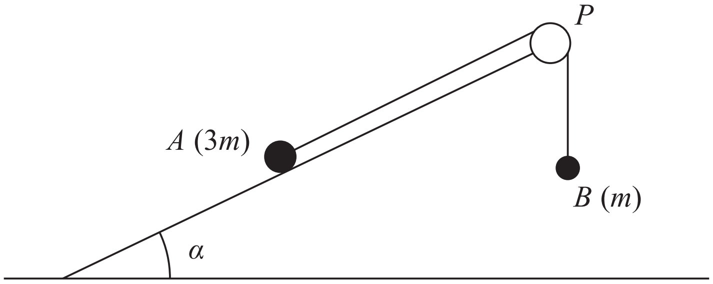


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

(c) sketch a velocity-time graph for the motion of B, from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(2)

In reality, the string is not light.

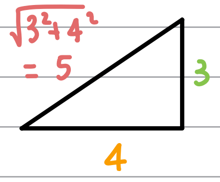
(d) State how this would affect the working in part (b).

(1)

P 6 8 8 2 4 A 0 4 2 0

Question 2 continued

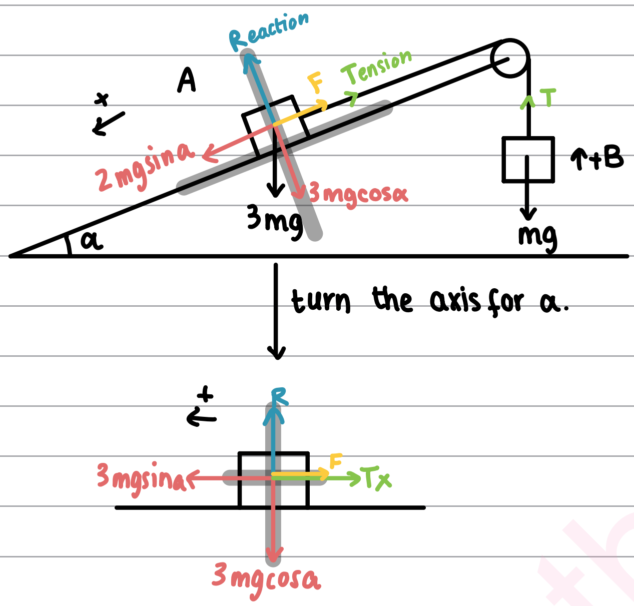
$\tan\alpha = \frac{3}{4}$



$\therefore \cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

Let's draw the forces



(a) Equation of motion for A:

Use $\Sigma F_x = ma$

Eq1 $3mg\sin\alpha - T - F = 3ma$ M1A1

(b) For R : $\Sigma F_y = 0$

$R = 3mg\cos\alpha = 3mg \times \frac{4}{5} = \frac{12}{5}mg$ A1

As it's moving $F_{max} = \mu R$

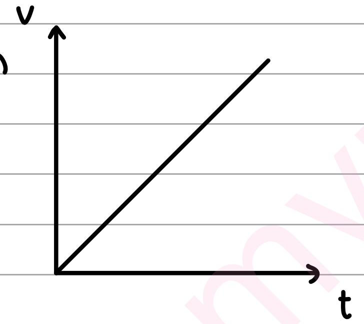
$F = \frac{1}{6} \times \frac{12}{5}mg = \frac{2}{5}mg$ B1

Equation of motion for B:

Use $\Sigma F_y = ma$

Eq2 $T - mg = ma$ M1A1

(c)



dB1

straight line graph since the acceleration is constant

Solve Eq1 and Eq2 simultaneously: dM1

$3mg\sin\alpha - T - F = 3ma$, $T = ma + mg$

$3mg \times \frac{3}{5} - (ma + mg) - \frac{2}{5}mg = 3ma$

$\frac{9}{5}mg - ma - mg - \frac{2}{5}mg = 3ma$ cancel m's

$\frac{9}{5}g - g - \frac{2}{5}g = 4a$

$4a = \frac{2}{5}g$

$a = \frac{1}{10}g$ hence shown A1

(d) Modelling Assumption "light string" ⟶ tension is the same in all sections of the string.

∴ if the string is <u>not</u> light → the tension for A would be different to the tension on B. B1

P 6 8 8 2 4 A 0 5 2 0

Turn over

Question 2 continued

Question 2 continued

(Total for Question 2 is 12 marks)

3.

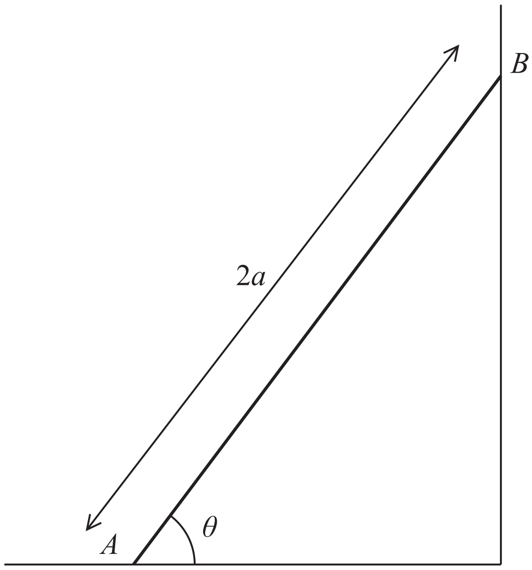


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

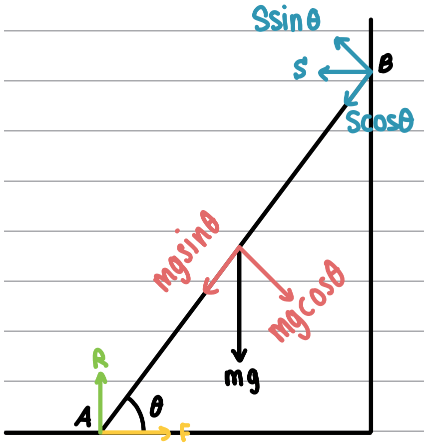
Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k.

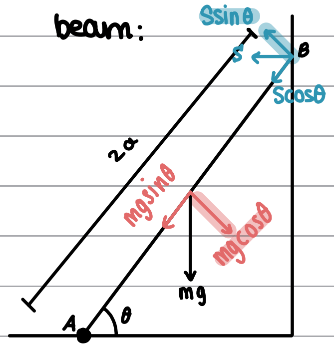
(5)

P 6 8 8 2 4 A 0 8 2 0

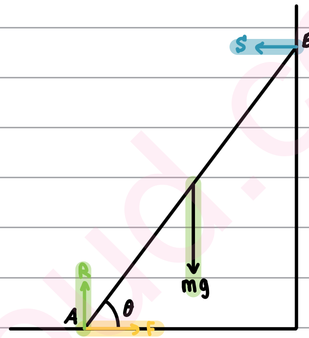
(a) Let's draw all the forces



$\overset{\curvearrowright}{\Sigma M_A} = 0$ as it's in equilibrium. For this equation we need the forces acting perpendicular to the beam:



Now lets use $\Sigma F_x = 0$ horizontally and $\Sigma F_y = 0$ vertically



$\overset{\curvearrowright}{\Sigma M_A} = a\,mg\cos\theta - 2a\,S\sin\theta$ M1

$a\,mg\cos\theta = 2a\,S\sin\theta$ A1

$F = S$ $\quad \Sigma F_x$

$R = mg$ $\quad \Sigma F_y$

B1

For the beam not to move, $F \leq \mu R$.

$F = S$, $\quad S = \dfrac{mg\cos\theta}{2\sin\theta}$. $\therefore F = \dfrac{mg\cos\theta}{2\sin\theta}$

$F \leq \mu R$, $R = mg$ $\quad \therefore \dfrac{mg\cos\theta}{2\sin\theta} \leq \mu mg$

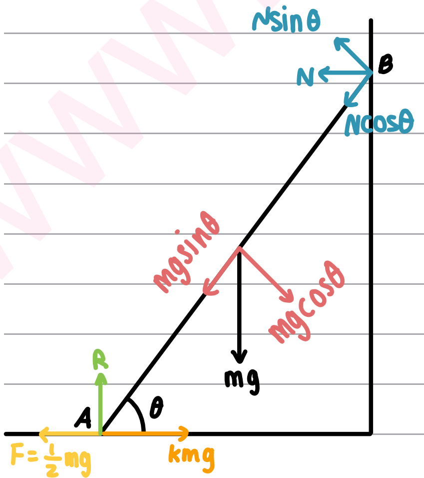
dM1

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

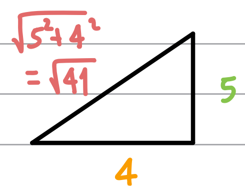
$\dfrac{1}{2} \times \dfrac{\cos\theta}{\sin\theta} \leq \mu$

$\therefore \mu \geq \dfrac{1}{2}\cot\theta$ as required A1

(b) Redraw the diagram:



$\tan\alpha = \dfrac{5}{4}$



$\therefore \cos\alpha = \dfrac{4}{\sqrt{41}}$

$\sin\alpha = \dfrac{5}{\sqrt{41}}$

Limiting equilibrium $\therefore F = \mu R$, $= \frac{1}{2}mg$ (from a)

Question 3 continued

$\Sigma M_A = 0$ (same as in (a), only forces perpendicular to the beam)

$$\Sigma M_A = amg\cos\theta - 2aN\sin\theta = 0 \quad \text{M1}$$

$$amg \times \frac{4}{\sqrt{41}} = 2aN \times \frac{5}{\sqrt{41}}$$

$$4mg = 10N \quad \text{A1}$$

Now lets use $\Sigma F_x = 0$ horizontally

ΣF_x: $N = kmg - F$ B1

Use the equations we found to get k: dM1

$$4mg = 10(kmg - F)$$

$$4mg = 10kmg - 10(\tfrac{1}{2}mg) \quad \text{cancel } mg.$$

$$4 = 10k - 5$$

$$9 = 10k$$

$$k = 0.9 \quad \text{A1}$$

P 6 8 8 2 4 A 0 1 0 2 0

Question 3 continued

(Total for Question 3 is 10 marks)

4.

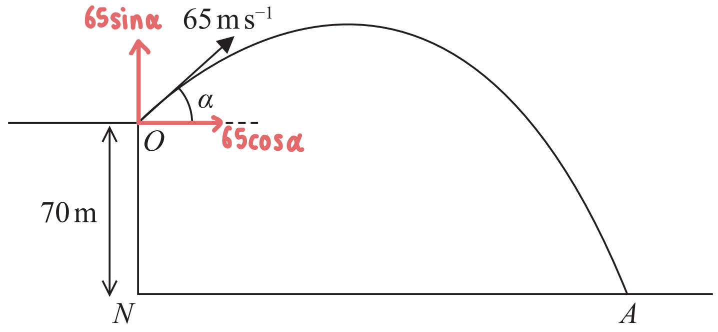


Figure 3

A small stone is projected with speed 65 m s^{-1} from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2} NOT 9.8 m s^{-2} !!!

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

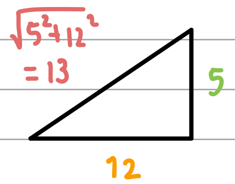
(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

$\tan\alpha = \frac{5}{12}$



$\therefore \cos\alpha = \frac{12}{13}$

$\sin\alpha = \frac{5}{13}$

Question 4 continued

(a) use vertical motion (↑+) - use suvat

$s = -70$

$u = 65\sin\alpha = 65 \times \frac{5}{13} = 25$

~~v~~

$a = -10$

$t = t$

Use formula $s = ut + \frac{1}{2}at^2$ M1

$-70 = 25t - \frac{10}{2}t^2$ M1 A1

$0 = 5t^2 - 25t - 70$

$0 = t^2 - 5t - 14$

$0 = (t-7)(t+2)$

A1 $t = 7$ ~~$t = -2$~~ Reject as time can't be negative

(b) To get the speed we need to find both the horizontal and vertical components.

Horizontal

$u = 65\cos\alpha$

$= 65 \times \frac{12}{13} = 60\ ms^{-1}$ B1

horizontal speed does not change!

Vertical

$s = -70$

$u = 65\sin\alpha$

$v = v$

$a = -10$

$t = 7$

Use Formula $v = u + at$

$v = 65 \times \frac{5}{13} - 70$ M1

$v = 25 - 70 = -45\ ms^{-1}$ A1

Use Pythagoras' Theorem to find the speed:

$\sqrt{(60)^2 + (-45)^2}$ M1

$= 75\ ms^{-1}$ A1

(c) · The value of g used is just an approximation

· The spin of the stone

· effect of wind

· dimension of the stone may affect motion B1

Turn over

P 6 8 8 2 4 A 0 1 3 2 0

Question 4 continued

P 6 8 8 2 4 A 0 1 4 2 0

Question 4 continued

(Total for Question 4 is 10 marks)

5. At time t seconds, a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\text{m s}^{-1}$

(6)

(a) To get the acceleration from the velocity we differentiate M1

$$v = 3t^{\frac{1}{2}}i - 2tj$$

$$a = \frac{dv}{dt} = \frac{3}{2}t^{-\frac{1}{2}}i - 2j \quad \text{A1}$$

(b) So we need to find t when the velocity is parallel to i-j

Get the $\frac{i}{j}$ ratio from the velocity equation + equate to $\frac{i}{-j}$:

$$\frac{3t^{\frac{1}{2}}}{-2t} = \frac{1}{-1}$$

$$3t^{\frac{1}{2}} = 2t \quad \text{M1}$$

↓ solve for t dM1

$$9t = 4t^2 \quad \text{square both sides}$$

$$0 = 4t^2 - 9t = t(4t-9) \rightarrow t = 0,\ t = \frac{9}{4}$$

$$\therefore t = \frac{9}{4} \quad \text{A1}$$

(c) To get the position from the velocity we integrate M1

$$a = \int v\,dt = \int 3t^{\frac{1}{2}}i - 2tj$$

$$= \frac{3}{\frac{3}{2}}t^{\frac{3}{2}}i - t^2j$$

$$= 2t^{\frac{3}{2}}i - t^2j + c \quad \text{A1}$$

★ Simple Integration:

$$\int x^n\,dx = \frac{1}{n}x^{n+1} + c$$

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P 6 8 8 2 4 A 0 1 6 2 0

Question 5 continued

Now to get c substitute $t=1$, $r=-j$:

$$-j = 2(1)^{\frac{3}{2}}i - (1)^2 j + c$$

$$-j = 2i - j + c$$

$$c = -2i.$$

$$\therefore \quad r = 2t^{\frac{3}{2}}i - t^2 j - 2i \quad \text{A1}$$

(d) We are told that $|v| = 10\,\text{ms}^{-1}$, so we can substitute into the Pythagoras' theorem and solve to get t:

$$\sqrt{(3t^{\frac{1}{2}})^2 + (2t)^2} = 10 \quad \text{M1}$$

$$9t + 4t^2 = 100 \quad \text{M1}$$

$$0 = 4t^2 + 9t - 100$$

$$0 = (t-4)(4t+25)$$

A1 $t = 4$ $\quad t = -\frac{25}{4}$ Reject, time can't be negative.

Now substitute $t=4$ into the position formula to get the vector OP:

$$r = 2(4)^{\frac{3}{2}}i - (4)^2 j - 2i$$

$$= 16i - 16j - 2i$$

$$= 14i - 16j \quad \text{M1}$$

use pythagoras' theorem to get the distance OP:

$$\sqrt{(14)^2 + (-16)^2} \quad \text{M1}$$

$$= \sqrt{452}$$

$$\downarrow$$

$$2\sqrt{113}\ \text{m} \quad \text{A1}$$

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 14 marks)

TOTAL FOR MECHANICS IS 50 MARKS